

QuasiCerenkov Radiation of Relativistic Electrons in Crystals in the Presence of External Excitations

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Abstract

The paper is devoted to the study of the influence of crystalline lattice distortions due to external excitations (acoustic vibrations, temperature gradient, etc.) on the Quasicerenkov radiation. Equations describing Quasicerenkov radiation of charged particles in distorted crystals are derived. These equations are solved numerically. It is shown that certain types of lattice deformations may intensify the Quasicerenkov radiation by several times.

It is known that being in uniform linear motion a charged particle radiates only if the Cerenkov condition is satisfied or the medium has space and/or time inhomogeneities. In the case of space inhomogeneities intensity of the radiated beam, its direction, and frequency are depended on the type and size of inhomogeneities. If these inhomogeneities are periodically arranged then the radiation, with the wavelength of the range of this periodicity, may be intensified due to the interference of the waves radiated from the different inhomogeneities. Such radiation takes place when a charged particle moves through a crystal. Radiation emitted under the Bragg angles is formed by diffraction of the secondary waves accompanying the charged particles on the crystalline lattice. It is called Quasicerenkov radiation [3] (QCR) (or Parametric x-ray radiation [2]). The QCR was predicted [1],[2] in 1971 and their properties have been studied on the basis of semi classical arguments in many theoretical works (for example see [3]). The experimental observation of QCR occurred after 1985 (see Refs [4]-[5]). The results of this and following experiments in the main agree with the theory. Recent works (see [6]) have revealed some disagreement of the measured and theoretical values of the ratios of higher order radiation intensities to first order radiation intensity for mosaic graphite.

The existing QCR theory was developed only for perfect (dynamic theory) and mosaic (kinematic theory) crystals that can not describe the dynamic effects in the presence of weak distortions in the crystal. In the present paper an attempt is made to develop a new method for investigating the influence of lattice distortions on QCR.

The QCR phenomenon is described by Maxwell's equations, where the permittivity of the medium is considered to be a periodic function of the spatial coordinates. The Fourier-transform of the electromagnetic induction with respect to the time is found to be of the form $\vec{D} = \vec{D}_s + \vec{E}_e$, where \vec{E}_e is the field of the moving charge in vacuum, and \vec{D}_s is the scattered field. Then the Maxwell equations are reduced to

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$$\Delta \vec{D}_s + \frac{\omega^2}{c^2} \vec{D}_s + \text{rotrot}(\chi \vec{D}_s) = -\text{rotrot}(\chi \vec{E}_e), \quad (1)$$

where $\chi(\vec{r}, \omega)$ is the polarizability of the medium, ω is the radiation frequency and for \vec{E}_e it is easy to find

$$\vec{E}_e = \frac{ie}{2\pi^2 v} \iint \frac{\gamma^{-2} \vec{k} + \vec{q}}{\gamma^{-2} k^2 + q^2} \exp(-i\vec{k}\vec{r} - i\vec{q}\vec{r}) d\vec{q}, \quad (2)$$

where e is the particle charge, γ is the Lorentz factor, $\vec{k} = \omega \vec{v}/v^2$, \vec{v} is the particle velocity, while integration is over the vectors perpendicular to the particle path ($\vec{q} \perp \vec{v}$). As it is seen from (2), the field of the moving charged particle can be interpreted as a sum of secondary waves with wave vectors $\vec{k} + \vec{q}$. When the crystal is distorted in a way that the characteristic length of the lattice deformations exceed many times the sizes of elementary cell then for $\chi(\vec{r}, \omega)$ we can write [9]

$$\chi(\vec{r}, \omega) = \sum_b \chi_b \exp(-i\vec{b}(\vec{r} - \vec{u})), \quad (3)$$

where summation is performed with respect to the reciprocal lattice vectors \vec{b} , \vec{u} is the vector displacement of the elementary cell from its initial position. Taking into account (3) it is convenient to search a solution of (1) in the form

$$\vec{D}_s = \frac{ie}{2\pi^2 v} \sum_g \iint \vec{D}_g \exp(-i\vec{k}_g \vec{r} + i\vec{g}\vec{u}) d\vec{q}, \quad (4)$$

where $\vec{k}_g = \vec{k} + \vec{q} + \vec{g}$ and \vec{D}_g are slow functions of the coordinates. Substituting (4) into (1) and using the principle of superposition, after multiplying both sides of the equation by $\exp(i\vec{k}_h \vec{r} + i\vec{h}\vec{u})$ and integrating over the elementary cell volume we obtain an infinite system of equations

$$2(\vec{k}_h \vec{\nabla}) \vec{D}_h + i\left(\frac{\omega^2}{c^2} - k_h^2(1 - \chi_0) + 2(\vec{k}_h \vec{\nabla})(\vec{h}\vec{u})\right) \vec{D}_h - i \sum_{b \neq h} \chi_{h-b} \vec{k}_h \times \vec{k}_h \times \vec{D}_b = i\vec{F}_h, \quad (5)$$

where

$$\vec{F}_h = \frac{\chi_h}{\gamma^{-2} k^2 + q^2} [\vec{k}_h, [\vec{k}_h, \gamma^{-2} \vec{k} + \vec{q}]],$$

and \vec{h} takes all the values of the reciprocal lattice vectors. When deriving these equations we neglect small terms of second and higher order (i.e., the terms containing second derivatives of the slow functions \vec{u} and \vec{D}_h , the production of first derivatives by one another, and by χ_h as for x-ray frequency range $|\chi| \sim 10^{-6}$). The left-hand side of system (5) completely coincides with the Takagi equations [9] for x-ray diffraction in distorted crystals. Let us consider the case,

when for a given frequency only two strong waves scattered in the directions \vec{k}_0 and \vec{k}_h exist. In this case only two equations remain in system (5) describing the amplitudes \vec{D}_0 and \vec{D}_h . After separating the radiation into the normal and coplanar polarizations one can find the separate systems of two equations for every kind of polarization

$$\begin{aligned} 2(\vec{k}_0 \vec{\nabla}) D_0^\alpha + i a_{00} D_0^\alpha + i a_{0h}^\alpha D_h^\alpha &= i F_0^\alpha, \\ 2(\vec{k}_h \vec{\nabla}) D_h^\alpha + i(a_{hh} - 2(\vec{k}_h \vec{\nabla})(\vec{h} \vec{u})) D_h^\alpha + i a_{h0}^\alpha D_0^\alpha &= i F_h^\alpha, \end{aligned} \quad (6)$$

where index $\alpha = \sigma, \pi$ indicates the polarization type (σ corresponds to the normal polarization when the amplitudes are perpendicular to the plane composed by wave vectors \vec{k}_0 and \vec{k}_h , and π corresponds to the coplanar polarization when they are in that plane),

$$a_{00} = \chi_0 k_0^2 - (\gamma^{-2} k^2 + q^2), \quad a_{hh} = \chi_0 k_h^2 - (\gamma^{-2} k^2 + q^2) + k_0^2 - k_h^2,$$

$$a_{0h}^\sigma = k_0^2 \chi_h, \quad a_{0h}^\pi = k_0^2 \chi_h \cos 2\theta, \quad a_{h0}^\sigma = k_h^2 \chi_h, \quad a_{h0}^\pi = k_h^2 \chi_h \cos 2\theta,$$

$\cos 2\theta = (\vec{k}_0 \vec{k}_h) / (k_0 k_h)$ and the vector amplitudes $\vec{D}_{0,h}$, $\vec{F}_{0,h}$ are defined by the scalar amplitudes $D_{0,h}^\alpha$, $F_{0,h}^\alpha$ by the expression

$$\vec{A}_{0,h} = (A_{0,h}^\sigma [\vec{k}_0, \vec{k}_h] + A_{0,h}^\pi [\vec{k}_{0,h}, [\vec{k}_0, \vec{k}_h]] / k_{0,h}) / (k_0 k_h \sin 2\theta),$$

where $\vec{A} = \vec{D}$ or \vec{F} . To solve the problem of finding a relativistic electron's QCR field it is necessary to specify boundary conditions for the system (6). For the Laue case of orientation the boundary conditions for the two wave approximation is

$$\vec{D}_0(\vec{r}_p) = \vec{D}_h(\vec{r}_p) = 0, \quad (7)$$

where \vec{r}_p is the radius vector of a point on the crystal entrance surface, as is no radiation field before the crystal. The number of γ -quanta with the energy $\hbar\omega$ emitted in the direction \vec{k}_h is

$$\frac{\partial N_h}{\partial \omega} = \frac{c \cos \theta}{4\pi \hbar \omega} \iint (|D_h^\sigma|^2 + |D_h^\pi|^2) dx dy, \quad (8)$$

where (x, y) are the coordinates of exit surface of the crystal. Equations (6) with the boundary conditions (7) may be analytically solved only for certain types of distortions of crystals. It should be mentioned that since these equations without right-hand side are the same as for the case of x-ray diffraction in the distorted crystal [9] and since the solution of inhomogeneous equations can be built by the solutions of homogeneous part of that equations then the problem of QCR is analytically solvable for each type of distortions for which the problem of x-ray diffraction is analytically solvable. For example they may be solved analytically in the case of quadratic deformations of crystalline lattice [8] (that is, in the case of temperature gradient or crystal bending). In common case they will be solved approximately by analytic or numeric methods. We have studied

obtained QCR equations by numeric methods for two practically interesting cases of the crystal's distortions described in [10]. In the first case

$$\vec{h}\vec{u} = \frac{2\pi u_0}{d} \sin\left(\frac{\pi z}{T}\right). \quad (9)$$

These type of distortions are generated when a piezocrystal is excited by an alternating voltage with the resonant frequency of the sample (the time dependence in (9) is omitted, as the time of particle transmission through the crystal is much less than the period of the acoustic vibrations). In the second case the crystal is heated on one side and is cooled on the other one, so that the direction of the temperature gradient is perpendicular to the reflection planes, and the function $\vec{h}\vec{u}$ has the form

$$\vec{h}\vec{u} = \frac{2\pi u_0}{d} \frac{4\pi z}{T} \left(1 - \frac{z}{T}\right). \quad (10)$$

In both cases the crystal is oriented by the symmetric Laue geometry when the diffraction vector is parallel to the entrance surface of crystal. In these cases $\vec{h}\vec{u}$ depends only on the coordinate perpendicular to the diffraction vector \vec{h} , and the equations (6) can be reduced to the system of ordinary differential equations. The calculations are carried out by the Runge-Kutta numeric method for the parameter values according to experimental data of [10]. The results for acoustic vibration case are presented in the Figs. Fig.1 shows the energy or the frequency dependence of the number of radiated γ -quanta emitted in the diffraction direction for various values of the amplitude of the acoustic vibrations. Fig. 2 shows the dependence of the integral number of the emitted QCR γ -quanta on the amplitude of the acoustic vibrations.

Fig 1. The frequency dependence of QCR radiated photons' number for different values of acoustic vibrations' amplitude ($\nu = 0$ is equivalent to $E\gamma = 10.1\text{KeV}$): a) $u_0/d = 0$; b) $u_0/d = 30$; c) $u_0/d = 60$ (d is the interplane distance).

Fig 2. The integral number of diffracted γ -quanta depended on the acoustic vibration's amplitude.

As it is seen from Figs, the QCR intensity increases several times with the increase of the amplitude of the acoustic vibrations. For the high values of the vibration amplitude the intensity curve goes to the saturation. The similar results are obtained in the case of temperature gradient. These results are in good agreement with the experimental results of [10] and [11].

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